CNS 187 - Neural Computation
Problem Sheet 2

Handed out:	7 Oct 00
Due:	13 Oct 00, 5pm

This week's homework explores integrate-and-fire neurons, the equivalence of various formulations of continuous-time continuous-activity neural networks, and introduces the final project. **You don't want to leave the final project until the end, because the *mus silicium* web server might be unresponsive if everyone tries to use it at once.**

1.1 Computing with Integrate & Fire Neurons (2 points)

We have seen in lecture that an interesting and versatile model for spiking in single neurons is the *integrate-and-fire* (I&F) model. In its simplest form, it assumes a neuron capacitively integrates current injected into its soma. The current might arise due to post-synaptic potentials due to spikes from other cells, or from an experimenter's electrode. Either way, we denote the net current coming into the cell by $I(t)$ and write,

$$C\frac{dV}{dt} = I(t) \qquad (1)$$

$$C\frac{dV}{dt} = -\frac{V}{R} + I(t) \qquad (2)$$

where the first equation is the perfect integration case (corresponding to the perfect I&F model), and the second equation adds a resistive leakage term (corresponding to the leaky I&F model). These equations hold *except* at the instant the cell fires: when the membrane voltage V reaches some threshold V_T, the cell fires an action potential (a "spike") and instantly resets V to zero. This description leads to the equivalent circuit shown below.

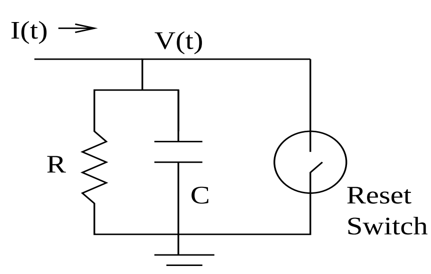

Figure 1: An integrate-and-fire circuit.

The "reset switch" symbol handles the nonlinear, discontinuous firing process. An additional modification is to have the switch clamp the voltage to 0 for a period t_{ref} after every spike;

t_{ref} is known as the *refractory period*, because the cell cannot spike during that time, and any input current which arrives during this interval, is shunted to ground.

Suppose the cell i produces spikes at times t_i^k. Let

$$f_i(t) = \sum_{k=1}^{\infty} \delta(t - t_i^k)$$

where $\delta(\cdot)$ is the Dirac delta function. Then the synaptic current input from cell i to cell j is $(\alpha_{ij} \star f_i)(t)$, where $\star$ indicates convolution. In the absense of external input to cell i,

$$I_i(t) = \sum_{j=1}^{N} (\alpha_{ij} \star f)(t).$$

Leaky integrate-and-fire systems, even the simplistic one described above, are difficult to analyze directly. It is difficult to even write the full dynamics for V explicitly in terms of familiar functions. However, the simulations are fairly straightforward.

1. Fill in the Euler integration line in `iafsim.m`.
2. Use the program `gensig.m` to create a one-second span of white-noise "stimulus waveform" $I(t)$ (1 msec $\leq t \leq$ 1000 msec) with slowly-varying input activity (*e.g.*, bandwidth = 10 Hz, mean = 0, standard deviation = 40pA). Run `iafsim` on this stimulus waveform. Because the I&F model is using "biologically plausible" units, make sure the waveform is properly scaled, according to the conventions of `iafsim.m`. Try four models: to model a vanilla integrate-and-fire model (with no leak conductance), use `R = Inf, C = 100pF, Tref = 0, Vt = 15mV`. Add leak by setting `R = 100Mohm`. Add a refractory period by additionally setting `Tref = 5msec`. Finally, also add noise by adding an independent, 300 Hz bandwidth, mean 0, standard deviation 100pA Gaussian current to the input signal (a deterministic step current). Hand in your plots.
3. Create four plots of spiking frequency vs input current, one for each model. Vary the constant input current from 0 to 500pA, or choose a reasonable range for your parameters.
4. Design a 2-input bitstream adder using leaky refractory integrate-and-fire units. Use synaptic currents of the form $\alpha_{ij}(s) = \{w_{ij} \text{ if } 0 < s < d; \text{ 0 otherwise } \}$. Input spikes arrive at times $\{0, \Delta t, 2\Delta t, \ldots\}$, and output spikes should occur at times $\{t_{delay}, t_{delay} + \Delta t, t_{delay} + 2\Delta t, \ldots\}$. You must show that the output spikes occur at exactly the right times (for some t_{delay} that you choose), but you don't have to simulate the network (unless you want to).

1.2 Network Equivalences (2 points)

In class, we derived the equation

$$\frac{dI_i}{dt} = -\frac{1}{\tau_i} I_i + \sum_{j=1}^{N} w_{ij} \sigma(I_j) + I_i^{inject}$$

as a mean-rate approximation for the behavior of a network of integrate-and-fire neurons. Here, I_i is the current flowing into cell i (as a function of time), τ_i is the time constant for all synaptic currents flowing into cell i, w_{ij} is the (positive or negative) strength of the synaptic current from cell j to cell i, I_i^{in} is a constant "external" input to cell i, and $\sigma(\cdot)$ is the function relating current input to mean firing rate. (I_i^{in} was not discussed in class, but it is a simple extension that will prove to be convenient. Also, here we allow each cell to have its own characteristic time constant.)

We also demonstrated a simple electrical circuit[1] whose behavior is described by the related equation

$$\frac{dV_i}{dt} = -\left(\frac{1}{C_i R_i} + \sum_{j=1}^{N} \frac{1}{C_i R_{ij}}\right) V_i + \sum_{j=1}^{N} \frac{s_i}{C_i R_{ij}} g(V_j) + i_i^{in},$$

where V_i is the voltage on the i^{th} unit (as a function of time), C_i is the capacitance of the i^{th} unit's capacitor, R_i is the i^{th} unit's resistance to ground, R_{ij} is the resistance between unit i and the (positive if $s_{ij} = 1$, negative if $s_{ij} = -1$) output of unit j, i_i^{in} is a constant "external" input to unit i, and $g(\cdot)$ is the transfer function of op-amps. (Here, we introduce i_i^{in} and we allow each unit to have its own capacitance and resistance to ground.)

1. Show that these two types of system are equivalent if $g(x) = 2 * \sigma(\delta x + \Delta) - 1$, that is, for any choice of constants $(\tau_i, w_{ij}, I_i^{in})$ in the mean-rate network, you can find a set of constants $(C_i, R_i, R_{ij}, s_{ij}, i_i^{in})$ and linear relation $V_i = \alpha I_i + \beta$ that transforms one equation into the other, exactly. Note that $s_{ij} \in \{-1, +1\}$ and $\tau_i, C_i, R_i, R_{ij} > 0$, but $w_{ij}, I_i^{in}, i_i^{in}$ may have any real value. To be concrete, you may use $g(x) = \tanh(x)$. Sketch $g(\cdot)$ and $\sigma(\cdot)$.

2. Another common formulation of continuous-valued neural networks uses the equation

$$\frac{dV_i}{dt} = -\frac{1}{\tau_i} V_i + \sigma\left(\sum_{j=1}^{N} w_{ij} V_j + i_i^{in}\right).$$

   Here, input from all other units is first summed, and then the non-linearity is applied. Show that, if w_{ij} defines an invertible matrix and $\tau_i = \tau_j = \tau$, then this formulation is also equivalent to the mean-rate equations. (Hint: this linear transformation *almost* works: $\mathbf{I} = \mathbf{W}\mathbf{V}$, where $\mathbf{I}, \mathbf{W}$, and $\mathbf{V}$ are the vectors and matrices corresponding to I_i, w_{ij}, and V_i.)

1.3 Final Project Plan (1 point)

The final project, based on the *mus silicium* contest proposed by John Hopfield and Carlos Brody[2], will be due at the end of the quarter. You do not have to submit anything to their

[1] If you wish, also see HKP 3.4 for a related, but not idential, circuit.

[2] `http://neuron.princeton.edu/~moment`

competition, but if you complete your assignment in time and believe it a viable contender, go for it[3]! For the projects you can work alone or in pairs. They will be due December 5th.

There are three varieties of the final project that are available.

I. Based on contest **A** from the *mus silicium* competition: Write a 5-page essay describing your hypothesis of how *mus silicium* operates, based on deductions made from experiments that you have done or those presented in the paper. Your grade will be based primarily on the reasoning and deductive process you use. E.g., do each of your (thought– or simulation–) experiments test a clear hypothesis, and get a clear answer? Is your reasoning sound? Do your clearly state your assumptions?

II. Based on contest **B** from the *mus silicium* competition: Design an artificial neural network, comparable to the *mus silicium*, which performs the same task. You are advised to examine the official contest rules[4] where more information about the precise task (input/output) is given, as well as clarification of words like "comparable." Your grade will be based primarily on how well you explain, and justify, the design of your network ("what did you do and why did you do it this way?") and how clearly characterize its performance ("on what inputs does it or doesn't it work, and why?"). You do not need to solve the competition in order to get a good grade.

III. Self-defined project: Investigate some aspect of the *mus silicium* project that interests you. We are open to any reasonable suggestions. For example, you could...

- ...make a neural network to perform the same task as *mus silicum*, but using *any* neuronal model you learned in class.
- ...compare the *mus silicium* (as it is described in the paper) to what is known about the neural substrate of biological hearing, and discuss the biological plausiblity of the *mus silicium* neuronal models.
- ...characterize the types of signal variation (noise, distortion, amplitude) to which the *mus silicium* response is invariant.
- ...write a paper about the problem of speech recognition, either from an engineering standpoint or a psychophysical standpoint.

Self-defined projects must be approved by the professor. Consider this week's assignment a project proposal. It will either be approved, or you will be asked to speak with the professor to define the project more clearly.

At different intervals throughout the quarter, you will be asked as part of your homework to indicate your progress on the final project.

[3]Everybody could use and extra 500 dollars around the holidays. However, you should talk to the professor or your TA's before submitting, as we have made special arrangements with Hopfield and Brody to insure they don't get swamped. Note that the competition entries are shorter than your final projects. Also note: the competition deadline is December 1st, several days *before* the final project is due.

[4]`http://shadrach.cns.nyu.edu/~carlos/Organism/Competition/index.html`

This week, your assignment is to read the paper[5], do an experiment[6] using your own `.wav` file, and plan your project. If you don't have a way to record `.wav` files, please come by BBB333 to make a recording. More information will be posted on the Announcements page.

Therefore, as part of problem set 2, please hand in...

1. ...a print-out of your sound's waveform and the spiking pattern of a *mus silicium* neuron. Tell us which neuron, and what your sound is.
2. ...your choice of topic and a one page description of how you plan to approach the problem. For example, you could list a few of the experiments you will perform, or describe how you plan to design your network. If this is a self-defined topic, state clearly the criteria for success (i.e. on what basis should we grade you?)
3. ...a schedule of your expected weekly progress. One sentence per week will be sufficient.

If you are working with a partner, you must state who your partner is, and both individuals must write their own descriptions and schedules (which don't have to agree). Halfway through the quarter, each individual or group will give a short presentation of their project and progress to the rest of the class at a dinnertime meeting. Pizza will be provided.

[5] `http://shadrach.cns.nyu.edu/~carlos/Organism/Docs/paper.pdf`
[6] `http://shadrach.cns.nyu.edu/~carlos/Organism/Experiments/addnew.html`