CNS 187 - Neural Computation
Problem Sheet 0 - Math Background

Handed out:	26 Sept 00
Due:	3 Oct 00 in class

This problem set is designed to give you an idea of the level of mathematics required for this class. While the following problems may seem like an eclectic mixture of topics, each problem was chosen because it explores a mathematical technique used frequently in this course.

Students should be proficient in all the topics included as problems on this sheet. While this sheet *isn't graded*, we intend to use it to identify those who will need to improve their mathematics to a level appropriate for the course. It is therefore a requirement that this problem set be completed.

This should not take you more than 3 hours to complete. Use this timeline to judge your own abilities.

0.1 Linear Algebra

For the two generalized matrices

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} \tag{1}$$

a) Write down $\mathbf{AB}$.
b) For general $\mathbf{A}$ of size $n \times l$ and $\mathbf{B}$ of size $l \times m$, write the component $\mathbf{AB}_{ik}$ in terms of a sum over the products of the elements of $\mathbf{A}$, a_{ij} and $\mathbf{B}$, b_{jk}.

If $\mathbf{A} = \begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$

c) determine the eigenvalues and eigenvectors of $\mathbf{A}$
d) what will happen to the eigenvalues and eigenvectors if the diagonal values are changed to 2 (equivalent to adding the matrix by 2*I, where I is the identity matrix).

0.2 Random Walk

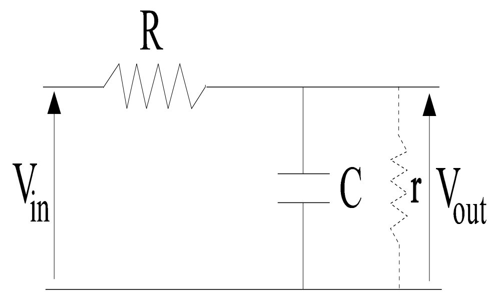
It's 2 AM and the Rathskeller has just closed for the night. N drunk graduate students and professors pile out of the bar onto the olive walk. At every step, each person stumbles one step of length Δ to the left or right along the sidewalk with equal probability*. After r steps, what is

a) the mean position of the members of the group?
b) their r.m.s. distance from the bar?

*Note: we assume here that there is no interaction between people, i.e. we don't consider them to be bumping into each other, rather they walk through or past each other on parallel paths.

0.3 RC Circuits

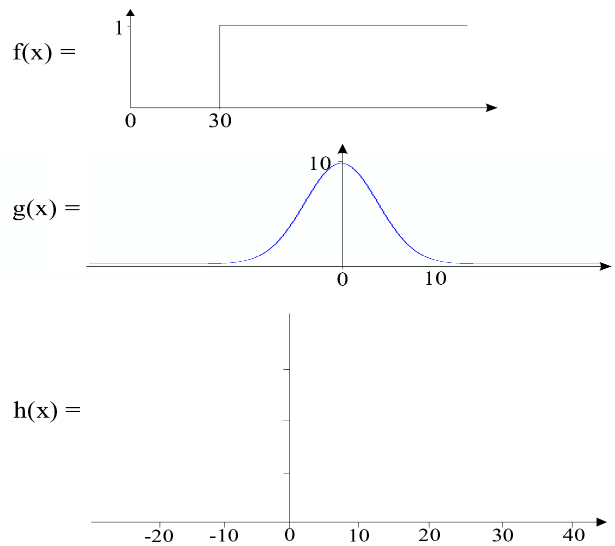
Consider the low-pass filter:



$$V_{in} = \begin{cases} 0V, & t < 0 \\ 5V, & t > 0 \end{cases} \tag{2}$$

a) Sketch the voltage response of the circuit as a function of time, with and without the parallel resistance, r.
b) Give the functional form of this response, again with and without the parallel resistance.

0.4 Convolution



Given the form of the functions $f(x)$ and $g(x)$ in Figure 2, sketch roughly the form of

$$h(x) = \int_{-\infty}^{+\infty} g(s) f(x-s) ds \tag{3}$$

on the outline above, indicating the relative amplitudes at $x = 0$, $x = 30$ and $x = +\infty$.

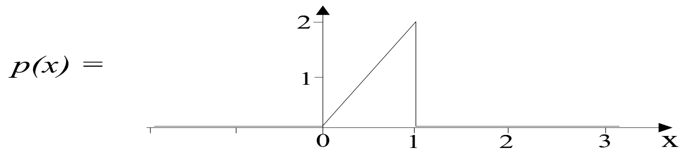
0.5 Steepest Descent

$$q(x, y, z, s, t) = \sin(x + y) + z + \frac{1}{2s + 4t} \tag{4}$$

If you are at $x = 1, y = 1, z = 1, s = 1, t = 1$, and you take a very small step of length L in some direction, in what direction should you take the step to decrease q the most?

0.6 Probability Density Functions

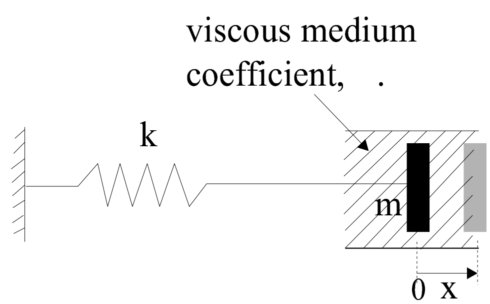
For the normalized probability density function, p(x), write an integral expression in terms of $p(x)$ for:



a) the mean, or expectation value, $\langle x \rangle$.
b) the variance, $\langle x^2 \rangle - \langle x \rangle^2$.
c) If $p(x)$ has the form shown in Figure 3, evaluate the mean and variance of x.

0.7 Simple Oscillating Systems

A horizontal, damped mass-on-spring oscillator is shown in Figure 4.



If the viscous damping force is given by $-\alpha \dot{x}$, the equation of motion of the system is given by

$$m\ddot{x} + \alpha\dot{x} + kx = 0, \tag{5}$$

$\alpha > 0$, $k > 0$.

a) By defining $v = \dot{x}$, rewrite (5) as a linear system of two first-order differential equations; i.e. identify the coefficients a_{ij} in the expression

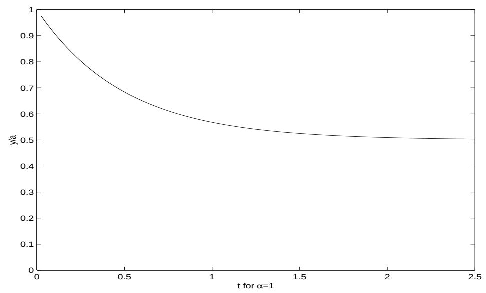
$$\dot{\mathbf{y}} = \mathbf{A}\mathbf{y} \text{ , where} \tag{6}$$

$$\begin{pmatrix} \dot{x} \\ \dot{v} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} x \\ v \end{pmatrix}. \tag{7}$$

b) Recall that (6) is solved by an expression of the form $c_1 e^{\lambda_1 t}\mathbf{v_1} + c_2 e^{\lambda_2 t}\mathbf{v_2}$. How are the scalars λ_i and the vectors $\mathbf{v}_i$ related to the matrix $\mathbf{A}$ that you found in (a)? Calculate the λ_i's, but don't trouble with the $\mathbf{v}_i$'s.

c) Assuming α and m to be fixed, what are the two ranges of the spring constant k for which *qualitatively* different solution behavior may be expected? With which range would you associate each of the following two plots (I. and II.) of x vs. t? Briefly explain your answer in terms of exponentials of real and imaginary numbers and physical intuition about the system shown in Figure 4.

I.



II.