**BEM 103**
**Introduction to Finance**
**Fall 2001/2**

# Homework 4

## Suggested Solutions

8.1 1. Solve for $I^u$ and $I^d$:

$$5 = I^u 10 + I^d 10,$$

$$4 = I^u 10 + I^d 6.$$

$I^u = I^d = 0.25.$

2.

$$P = I^u 20 + I^d 25 = 45/4 = 11.25.$$

9.2 Consider two portfolios. A = invest the present value of $K$ and shortsell the stock; B = buy one put. Repeat the analysis on p. 82-3 and show that payoff(A) $\leq$ payoff (B), whence value(A) = $K/(1+r)^{T-t} - S_t \leq$ value(B) = $P_t$.

9.4 Repeat the analysis of the previous answer but delete the stock in portfolio A. Then: payoff(A) $\geq$ payoff (B), whence value(A) = $K/(1+r)^{T-t} \geq$ value(B) = $P_t$.

9.5 By waiting, one can make *at most* 50 (= $K$). The present value of this is $50/(1+0.05) = 47.62$. By exercising right now, one makes more: $50 - 0.10 = 49.90$. So, exercise now.

9.9 Ms. Johnson has two options: to exercise (TE) or not to exercise (NE). Under TE, she will hold the following after the stock goes ex: Ex-dividend stock price $S_t$ minus strike price $K$ plus dividend $D$ of \$2. Under NE, she holds: option worth at least $S_t - K/(1+r)^{T-t}$. She should prefer NE, because the difference in payoffs between NE and TE is positive:

$$\begin{aligned} S_t - K/(1+r)^{T-t} - (S_t - K + D) &= K(1 - 1/(1+r)^{T-t}) - D \\ &= 55(1 - 1/1.1^{1/2}) - 2 \\ &= 0.56 \\ &> 0. \end{aligned}$$

14.4 In the up-state ($S_T = 150$), the payoff on the suggested portfolio is

$$-3 \times 50 + 2 \times 150 - 140(1+r) = 150 - 140(1+r).$$

In the down-state ($S_T = 75$), the payoff is:

$$2 \times 75 - 140(1 + r) = 150 - 140(1 + r).$$

(Note that the payoffs the the two states are the same: the portfolio is riskfree.) Evidently, the value of this portfolio is:

$$-3 \times 20 + 2 \times 100 - 140 = 0.$$

This should be the present value of the payoff, which implies that $r$ solves the following equation:

$$0 = \frac{1}{1 + r}[150 - 140(1 + r)].$$

Hence, $r = 7.14\%$.

14.5 Determine the state-price probabilities from the pricing of the stock:

$$96 = \frac{1}{(1 + .1)^{5/52}}[p120 + (1 - p)95].$$

$p = .0754$. Hence, the price of the call is:

$$C_t = \frac{1}{(1 + .1)^{5/52}}[(.0754)(120 - 112)] = .5977.$$

Since each call gives you the right to purchase 100 shares at the exercise price, the value of the call contract is 59.77

14.6 Do the same as in the previous answer. $p = .5643$, whence

$$C_t = \frac{1}{(1 + .08)^{1/12}}[(.5643)(42 - 39)] = 1.6821.$$

15.3

1. First determine the state-price probabilities for each of the branchings. These will be the same no matter what the branchings (check this!), namely, they solve:

$$9000 = \frac{1}{1.2^{1/2}}[p(9000)(1.25) + (1 - p)(9000)(0.8).$$

Hence, $p = .6565$. Now compute the tomorrow's value of the call. In the "up" state:

$$C^u_{t+1} = \frac{1}{1.2^{1/2}}[(.6565)((9000)(1.25)^2 - 9000) + (.3435)((9000)(1.25)(0.8) - 9000)] = 3034.$$

You can buy the option for 1500, which means you make 3034-1500=1534 in the "up" state. In the "down" state, the option is worth zero. Hence, you won't exercise your right to buy the option in that state. The present value of the option to buy the option for 1500 is:

$$C_0 = \frac{1}{1.2^{1/2}}[(.6565)(1534)] = 919.$$

2. If you don't have the option not to buy the option after 6 months, you lose in the "down" state a total of 1500. The present value now becomes:

$$C_0 = \frac{1}{1.2^{1/2}}[(.6565)(1534) - (.3435)(1500)] = 449.$$