## BEM 103

## Introduction to Finance

Fall 2001/2

## Homework 3

## Suggested Solutions

7.1 (No graphics here) In matrix notation ( $\rho=$ correlation between two asset returns):

$$
\begin{gathered}
\mu=\left[\begin{array}{c}
.125 \\
.16
\end{array}\right] \\
\Delta=\left[\begin{array}{cc}
.15^{2} & \rho(.15)(.2) \\
\rho(.15)(.2) & .2^{2}
\end{array}\right] .
\end{gathered}
$$

Then, for a vector of weights $x$ :

$$
\begin{aligned}
& E\left[r_{p}\right]=x^{\prime} \mu \\
& \sigma_{p}=\sqrt{x^{\prime} \Delta x}
\end{aligned}
$$

E.g., $\rho=0, x^{\prime}=[.25 .75]:$

$$
\begin{gathered}
E\left[r_{p}\right]=.15 \\
\sigma_{p}=\sqrt{.0239}=.155
\end{gathered}
$$

7.2

$$
E\left[r_{i}\right]=r_{F}+\beta_{i}^{M}\left(E\left[r^{M}\right]-r_{F}\right)=.05+0.5(.14-.05)=.095
$$

7.5 To apply the formula in the first answer, set

$$
\begin{gathered}
\mu=\left[\begin{array}{c}
.10 \\
.15 \\
0.05
\end{array}\right] ; \\
\Delta=\left[\begin{array}{ccc}
.05^{2} & \rho(.05)(.2) & 0 \\
\rho(.05)(.2) & .2^{2} & 0 \\
0 & 0 & 0
\end{array}\right] .
\end{gathered}
$$

With $\rho=0.25$ and $x^{\prime}=[.25$. 25 0.50],

$$
\begin{gathered}
E\left[r_{p}\right]=.0875 \\
\sigma_{p}=\sqrt{.0030}=.055
\end{gathered}
$$

7.9 1. None dominates or is dominated in mean-variance space.
2. a) $x^{\prime}=[.5$. 50$]$. Then: $E\left[r_{p}\right]=.08$ and $\sigma_{p}=.05$.
b) $x^{\prime}=\left[\begin{array}{lll}.5 & 0 & .5\end{array}\right]$. Then: $E\left[r_{p}\right]=.13$ and $\sigma_{p}=.19$.
c) $x^{\prime}=[0.5 .5]$. Then: $E\left[r_{p}\right]=.15$ and $\sigma_{p}=.20$.

None dominates or is dominated.
7.10 First compute cost-of-capital. Firmwide:

$$
E[r]=.05+(1.3)(.1)=.18
$$

Division P:

$$
E[r]=.05+(1 .)(.1)=.15
$$

Division E:

$$
E[r]=.05+(1.5)(.1)=.20
$$

Within a one-period (two-date) context, IRR and NPV produce the same decisions (unless there is no IRR). Decisions are as follows:

1. Company-wide: accept none.
2. Division P: accept none; Division E: accept none.
7.11
3. discount rate is

$$
.06+(1.15)(.1)=.175
$$

2. First determine the beta of the entire firm $\left(\beta_{V}\right)$. Let $B$ be the value of debt and $E$ be the value of equity. Note: $B / E=0.25$, so $E / B=4$. Betas are additive (like expectations), so

$$
\beta_{V}=\frac{B}{B+E} 0.3+\frac{E}{B+E} 1.15=\left(\frac{1}{1+E / B}\right) 0.3+\left(\frac{1}{1+B / E}\right) 1.15=(0.2)(0.3)+(0.8)(1.15)=0.98
$$

The discount rate is

$$
.06+(.98)(.1)=.158
$$

