BEM 103

Introduction to Finance

Fall 2001/2

Homework 1

Suggested Solutions

5.1 From the formula

$$P_t = \frac{1}{(1 + r_t)^t},$$

solve for the t-period spot interest rate. This gives: $r_1 = 0.05$, $r_2 = 0.05$, $r_3 = 0.06$. So,

$$PV = 100(.95 + .90 + .85) = 149.$$

5.2 Monthly interest payments in the case of quarterly compounding are (per dollar principal):

$$[(1 + 0.10/4)^4 - 1]/12 = 0.00865,$$

which is less than in the case of annual compounding:

$$[(1 + 0.105) - 1]/12 = 0.00875.$$

So, the quarterly compounding is more advantageous.

5.5 Solve the following system for the vector of pure discount bond prices:

$$\begin{bmatrix} 1043.29 \\ 1220.78 \\ 995.09 \end{bmatrix} = \begin{bmatrix} 100 & 1100 & 0 \\ 200 & 1200 & 0 \\ 80 & 80 & 1080 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix}.$$

The solution is:

$$\begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} .9091 \\ .8658 \\ .7899 \end{bmatrix}.$$

Find spot interest rates as in the first problem: $r_1 = 0.100$, $r_2 = 0.075$, $r_3 = 0.082$.

5.6 In two years, after it goes "ex dividend," the stock is worth (see formula on top of p. 30):

$$PV(2) = \frac{10(1.02)(1.10)^2}{.10 - .02} = 154.28.$$

So, today's stock price is

$$PV(0) = \frac{10(1.10)}{(1 + 0.10)} + \frac{10(1.10)^2 + 154.28}{(1 + 0.10)^2} = ~~128.50.~~ \text{ } 147.50$$

5.8

$$PV = \frac{X_1}{1+r}\sum_{t=1}^{\infty}\left(\frac{1+g}{1+r}\right)^{t-1} = \frac{X_1}{1+r}\frac{1}{1-\frac{1+g}{1+r}} = \frac{X_1}{r-g}.$$

5.9 This annuity can be obtained as a portfolio of a long perpetuity starting now and a short perpetuity starting at $T+1$. Hence,

$$PV = +\frac{X}{r} - \frac{1}{(1+r)^{T+1}}\frac{X}{r}.$$

5.12 (a) The amount needed now equals: $55000./(1+0.05)^4 = 45249$.

(b) The payment A needed each year solves:

$$A[1.05 + 1.05^2 + 1.05^3 + 1.05^4] = 55000,$$

i.e., $A = 12153$.

6.3 Strategy 1 (repeated 4 times):

$$PV = -47(1 + \frac{1}{1.1^2} + \frac{1}{1.1^4} + \frac{1}{1.1^6}) = -144.47.$$

Strategy 2 (repeated twice):

$$PV = -90(1 + \frac{1}{1.1^4}) = -151.47.$$

Strategy 3:

$$PV = -300.$$

Strategy 1 has the highest PV, so choose it.

6.5 Cash flows C_t:

$$C_0 = -240 - 60,$$
$$C_1 = C_2 = (200-100)(1-0.4) + \frac{240}{3}(0.4),$$
$$C_3 = C_2 + 40(1-0.4) + 60.$$

Hence,

$$\begin{aligned} NPV &= C_0 + C_1\frac{1}{1.08} + C_2\frac{1}{1.08^2} + C_3\frac{1}{1.08^3} \\ &= 4. \end{aligned}$$

(NPV is positive, so project is OK.)

6.6 Accounting break-even x solves the following equation:

$$-\frac{2000}{5} - 1500 + x(1.5 - 0.5) \stackrel{!}{=} 0.$$

Hence, $x = 1900$. In contrast, NPV break-even y solves the following. Cash flows C_t are (note use of accounting profits to compute taxes):

$$C_0 = -2000,$$

$$C_t = y(1.5 - 0.5) - 1500 - 0.34[(y(1.5 - 0.5) - 1500 - \frac{2000}{5}] = y(0.66) - 854,$$

$t = 1, 2, ..., 5$. y is the solution to:

$$-2000 + (1/1.16 + 1/1.16^2 + 1.16^3 + 1/1.16^4 + 1/1.16^5)(y(0.66) - 854) \stackrel{!}{=} 0.$$

Hence, $y = 2219$.

6.7 (a) Headache Pain Reliever: $C_0 = -10.2$ (all numbers in million dollars); cash inflow at t=1, 2, 3: $(4 - 1.50)5 = 12.5$; taxes: 12.5 @34% = 4.25; tax "rebate"because of depreciation (straight-line): 10.2/3 @34% = 1.156 (the tax "rebate" is a NOMINAL cash flow, so will have to be discounted at nominal rate!). Summary, split in real and nominal parts::

- Real cash flows: $C_{r0} = -10.2$, $C_{r1} = C_{r2} = C_{r3} = 8.25$.
- Nominal cash flows: $C_{n0} = 0$, $C_{n1} = C_{n2} = C_{n3} = 1.156$

Real interest rate = 13%; nominal interest rate ; (13+5)% = 18%

Altogether, discounting real cash flows at real rates and nominal ones at nominal rates (see below for a completely nominal treatment of the case):

$$\begin{aligned} NPV &= -10.2 + 8.25\left(\frac{1}{1.13} + \frac{1}{1.13^2} + \frac{1}{1.13^3}\right) + 1.156\left(\frac{1}{1.18} + \frac{1}{1.18^2} + \frac{1}{1.18^3}\right) \\ &= -10.2 + 8.25(2.3612) + 1.156(2.1743) \\ &= 11.793. \end{aligned}$$

(b) Broad pain reliever: $C_0 = -12$; cash inflow at t = 1, 2,3: $(4 - 1.70)10 = 23$; taxes: 23 @34% = 7.82; tax "rebate" on depreciation (again, NOMINAL): 12/3 @34% = 1.36; salvage value: 1, taxed @ 34%

Summary, split in real and nominal parts:

- Real cash flows: $C_{r0} = -12$, $C_{r1} = C_{r2} = 15.18$; $C_{r3} = 15.18 + 1(1 - 0.34) = 15.84$
- Nominal cash flows: $C_{n0} = 0$, $C_{n1} = C_{n2} = C_{n3} = 1.36$

Altogether,

$$\begin{aligned} NPV &= -12 + 15.18\left(\frac{1}{1.13} + \frac{1}{1.13^2}\right) + 15.84\frac{1}{1.13^3} + 1.36\left(\frac{1}{1.18} + \frac{1}{1.18^2} + \frac{1}{1.18^3}\right) \\ &= -12 + 15.18(1.6681) + 15.84(0.6931) + 1.36(2.1743) \\ &= 27.258 \end{aligned}$$

Conclusion: go for (b)

Note: If you use nominal cash flows throughout, numbers change little. For the first project, the NPV becomes:

$$\begin{aligned} NPV &= -10.2 + 8.25\left(\frac{1.05}{1.18} + \frac{1.05^2}{1.18^2} + \frac{1.05^3}{1.18^3}\right) + 1.156(2.1743) \\ &= 12.000 \end{aligned}$$

The difference is due to: $\frac{1.05}{1.18} = \frac{1}{1.1238} \neq \frac{1}{1.13}$.