1. Cashflows $t = 0 \quad 1 \quad 2 \quad 3$

without taxes yet

	$t=0$	1	2	3
$240/3 = 80 \rightarrow$	cost -240	dep. 80	dep. 80	dep. $80 + 40$
sales 200 $-$ costs 100 $\rightarrow$		100	100	100
	capital -60			60

NPV = PV − cost

$$= \frac{(\overset{20}{100-80})(1-\overset{tax}{0.4})+80}{(1+\underset{dis.\,rate}{0.08})} + \frac{(\overset{20}{100-80})(1-0.4)+80}{1.08^2}$$

$$+ \frac{(\overset{60}{100+40-80})(1-0.4)+80+60}{1.08^3} - (240+60)$$

$\approx \boxed{3.77}$

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2. $V_L = V_U + PV(\text{debt tax shield})$

Taxable Income $t =$	1	2	3
before debt tax shield	20	20	60
after " " " potential worth 25	0	0	35
		$\min(0, 20-25)$	$60-25$

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Debt Tax Shield $\quad 20(0.4) \quad 20(0.4) \quad 25(0.4)$

$$NPV \approx 3.77 + \left(\frac{20}{1.08} + \frac{20}{1.08^2} + \frac{25}{1.08^3}\right)0.4 \approx \boxed{25.97}$$

same with NPV =

$$\frac{100}{1.08} + \frac{100}{1.08^2} + \frac{(100+40-80-25)(0.6)+80+60+25}{1.08^3}$$

$-300 \approx 25.98$ ✓

3. $u = 1.5$, $d = 1/1.5 \approx 0.67$; $r = 0.05$ per period

Stock Price

$t = 0$	1	2
		225
	150	
100		100
	$\frac{200}{3} \approx 66.67$	
		$\frac{400}{9} \approx 44.44$

For each pair of binomial tree branches,

$$p^u = \frac{(1+r)^{T-t} - d}{u-d} = \frac{1.05 - (1/1.5)}{1.5 - (1/1.5)} \approx 0.46 \checkmark\text{'s with \#'s in tree}$$

Compound Option

$t = 0$	1	2
		$CO^{uu} = \max(\overset{\text{call}}{225-100}, \overset{\text{put}}{100-225}) = 115$
	$CO^u = PV^u(\text{call}/\cancel{\text{put}})$	
$CO_0 = ?$		$CO^{ud} = \max(100-100, 100-100) = 0$
	$CO^d = PV^d(\cancel{\text{call}}/\text{put})$	
		$CO^{dd} = \max(\cancel{44-100}, 100-44.44) = 55.56$

$$CO^u = \frac{1}{1.05}\left(0.46(115) + 0\right) \approx 50.38095238$$

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$$CO^d = \frac{1}{1.05}\left(0 + (1-0.46)(55.56)\right) \approx 28.57142857$$

$$CO_0 = \frac{1}{1.05}\left(0.46(50.38) + 0.54(28.57)\right) \approx \boxed{\$36.77}$$

4. $1000 \begin{cases} 1500 \\ 800 \end{cases}$ $\quad 1000 = \frac{1}{1.05}[p^u 1500 + (1-p^u)800]$

$$\frac{1500}{1.05}p^u - \frac{800}{1.05}p^u = 1000 - \frac{800}{1.05}$$

$r = 5\%$ $\quad$ 100 shares

$D = 10$ for one Cbond $\quad$ E & Cbonds 1:1

risk free land

$$p^u = \left(1000 - \frac{800}{1.05}\right) / \left(\frac{1500}{1.05} - \frac{800}{1.05}\right) \approx 0.357 \quad 1428571$$

$$B = \frac{1}{1+r}D - \frac{1}{1+r}E^*[\overset{\text{put}}{\max(D - V', 0)}] + \frac{1}{1+r}E^*[\overset{\text{call}}{\max(\tfrac{V'}{2} - D, 0)}]$$

Value of 100 convertible bonds = corporate debt B

$$\hookrightarrow \frac{1}{1.05}(100 \times 10) - \frac{1}{1.05}[(1-0.357)(1000-800) + (\cancel{p^u} \overset{800-1000<0}{0})]$$

will not convert $\hookrightarrow$ $\cancel{+\frac{1}{1.05}[0.357(0) + 0]} \Rightarrow 0$ b/c $1000 > \frac{1500}{2} = 750$ $> \frac{800}{2} = 400$

≈ 829.93

ONE Cbond $\rightarrow \hookrightarrow \div 100 \approx 8.30$

$V = B + E$; $E = V - B$

$$E = 1000 - 829.93 = \boxed{170.07} = \frac{1}{1.05}[0.357(\overset{1500-1000}{500}) - 0]$$

$$\boxed{\begin{array}{l} B = 829.93 \text{, one convertible bond} \approx 8.30 \\ E = 170.07 \text{, one share of equity} = 1.70 \end{array}}$$

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* assume personal wealth includes stock value held

* Please see description of scenarios to see what I thought problem meant

5(a) Scenario ①: firm pays dividends, you keep all stocks

$$\underbrace{(27\times 100)}_{\text{stock held}}+\underbrace{(3\times 100)}_{\text{dividends}}\underbrace{(1-0.28)}_{\text{taxes, income}} = 2,916$$

Scenario ②: firm does not pay div, you sell 10 shares

$$\underbrace{(30\times 90)}_{\text{stock held}}+\underbrace{(10\cdot 30)}_{\text{stock sold}}-\underbrace{(10(30-20))(0.28)}_{\text{capt. gains tax}} = 2,972$$

div. paid:	\$2,916
you sell 10 shares:	\$2,972

✓

(b) ① $2700+(300)(1-0.4) = 2880$

② $2700+300-(100)(0.16) = 2984$

div. paid:	\$2,880
you sell:	\$2,984

✓

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Q5 (Note: capital gains taxes are not paid and need not be taken into account until you realize them; in principle, you NEVER realize capital gains).

(a) Scenario 1:

$$300(1-0.28) + 100 \cdot 27 = 2916$$

Scenario 2:

$$-(300-200)(0.28) + 300 + 90 \cdot 30 = 2972$$

(b) Scenario 1:

$$300(1-0.40) + 100 \cdot 27 = 2880$$

Scenario 2:

$$-(300-200)(0.16) + 300 + 90 \cdot 30 = 2984$$

Q6 \$20. All the NPV goes to the shareholders.

Q7. Several possibilities:

(a) Firm A and B merge.

(b) Firm B takes over one division of Firm A with debt obligations in exchange of equity in Firm B.

(Note: issuing new debt is not an option, because the first coupon is not due until the future).

Q8 Over 1 year:

$$E(\ln W_1 - \ln W_0) = 0.15$$

$$V(\ln W_1 - \ln W_0) = 0.15^2 = 0.0225$$

Over 15 years:

$$E(\ln W_{15} - \ln W_0) = E\left(\sum_{t=1}^{15} \ln R_t\right)$$

$$= \sum_{t=1}^{15} E(\ln R_t)$$

$$= 15\,(0.15) = 2.25$$

$$V(\ln W_{15} - \ln W_0) = V\left(\sum_{t=1}^{15} \ln R_t\right)$$

$$= \sum_{t=1}^{15} V(\ln R_t) \qquad \text{(independence)}$$

$$= 15\,(0.15^2) = 0.3375$$

Q9 (a) $= P\left(\frac{W_1}{W_0} > 1\right) = P(\ln W_1 - \ln W_0 > 0)$

$$= P(\ln R_1 > 0)$$

$$= P\Big(\underbrace{\frac{\ln R_1 - 0.15}{0.15}}_{\sim N(0,1)} > -1\Big)$$

$$= 0.84$$

(b) $= P\left(\frac{W_{15}}{W_0} > 1\right) = P(\ln W_{15} - \ln W_0 > 0)$

$$= P\left(\sum_{t=1}^{T} \ln R_t > 0\right)$$

$$= P\Big(\underbrace{\frac{\sum_{t=1}^{T} \ln R_t - 2.25}{\sqrt{0.3375}}}_{\sim N(0,1)} > \underbrace{-\frac{2.25}{\sqrt{0.3375}}}_{-3.8730}\Big)$$

$$= 0.9998$$

[redacted]

(b) > (a)

q10 $(a) = P\left(\frac{W_1}{W_0} < 0.15\right) = P\left(\frac{\ln R_1 - 0.15}{0.15} < \frac{\ln 0.15 - 0.15}{0.15}\right)$

$= P\left(\frac{\ln R_1 - 0.15}{0.15} < -13.65\right)$

$(b) = P\left(\frac{W_{15}}{W_0} < 0.15\right) = P\left(\frac{\sum_{t=1}^{15} \ln R_t - 2.25}{\sqrt{0.3375}} < \frac{\ln 0.15 - 2.25}{\sqrt{0.3375}}\right)$

$= P\left(\frac{\sum_{t=1}^{15} \ln R_t - 2.25}{\sqrt{0.3375}} < -7.139\right)$

So, (b) > (a)

(Note: (a) and (b) are small numbers, but don't be fooled - they matter, e.g. when you have logarithmic utility - as Bernoulli suggested!)