

California Institute of Technology

BEM 103

Fall 2001-02

Final

Open book, open notes, 3-hour exam. Deadline: Friday, 14 December at noon. Questions are equally weighted.

1. A project has a cost of 240. It will have a life of 3 years. The cost will be depreciated straight-line to a zero salvage value, and is worth 40 at that time. Cash sales will be 200 per year and cash costs will run 100 per year. The firm will also need to invest 60 in working capital at year 0. The appropriate discount rate is 8%, and the corporate tax rate is 40%. What is the project's NPV?
2. In the previous question, what is the project's NPV if it is partly financed through a three-year debt issue with annual coupon equal to 25?
3. The current price of a stock in HS is \$100. Each period the stock price either moves up by a factor $u = 1.5$ or down by a factor $d = 1/u$. The per-period interest rate is 5%. Consider the following compound option written on HS. It gives you the right to choose among two options at $t = 1$:
 - A call option on HS with strike 100 and expiration $t = 2$,
 - A put option on HS with strike 100 and expiration $t = 2$.

Compute the value of this compound option at $t = 0$.

4. Suppose a firm's end-of-period value is either 1500 with probability 0.6 or 800 with probability 0.4. Today's firm value is 1000. The riskfree rate is 5%. The firm has 100 shares of equity and 100 convertible bonds with a face value of 10 each. A convertible bond can be converted in

shares of equity at the ratio 1:1. The bond pays no coupon. Compute the value of the convertible bond and of the equity.

5. A given share sells for \$30 just before it goes “ex.” If the firm pays a \$3 dividend per share, the share price is known to drop immediately afterwards to \$27. Suppose you own 100 shares. If the firm had not decided to pay a dividend, you would have had to sell 10 shares to receive the same (pre-tax) cash inflow of \$300. Assume that you originally bought the shares for \$20 each.
 - (a) If both ordinary personal tax rates and capital gains tax rates are 28%, what is your after-tax wealth under the two alternative scenarios?
 - (b) Suppose that tax rates for capital gains are lower: you pay 40% tax on ordinary income (including dividends), but only 16% on capital gains. What is your after-tax wealth under the two alternative scenarios?
6. A project has a NPV of 20. The owners decided to finance it partly through a debt issue with face value 100. The owners provided the remaining funds to start the project. In return, they get equity. The owners immediately sold their equity shares in the market. Can you determine how much money they will have made?
7. Firm A consists of two equally-sized divisions. Its value is \$100 and it is 30% debt financed ($V = 100$, $B = 30$). It pays a \$5 coupon on its debt. It has no taxable earnings. Firm B operates in the same sector. It has the same value as firm A, but is entirely equity-financed. It does have taxable earnings, of \$10, which are taxed at 20%. Suggest a strategy that enhances the total value of the two firms.
8. Consider investing \$1 in a stock market index (e.g., the S&P 500 index). Let R denote the one-year return (end-of-year wealth over beginning-of-year wealth) on the index. Assume that $\ln R$ is normally distributed with mean 0.15 and standard deviation 0.15. Over time, realizations of $\ln R$ are independent. What is the expected percentage change in

your wealth over one year (measured as $\ln W_1 - \ln W_0$, where W_t is your wealth at time t , and $W_0 = 1$)? What is the variance? How do these answers change if your horizon is T years, say $T = 15$, instead of 1? I.e., what is the expected percentage change in your wealth over 15 years and what is its variance?¹

9. Is the probability that you end up with more than you started (remember, $W_0 = 1$) higher or lower over one year than over 15 years?²
10. Is the probability that you end up with less than 15% of your initial wealth higher or lower over one year than over 15 years?

¹Remember that the CAPM is based on the notion of *diversification*: one can generally obtain the same mean return but less volatility by combining securities into portfolios. Questions 8 to 10 attempt to extend the idea of diversification to the time domain. The question is whether one can “diversify over time,” i.e., reduce risk by letting good years “cancel out” bad years. Your answers will give you an idea to what extent diversification over time works. Most stockbrokers implicitly think it does, because they advise younger people with longer investment horizon to invest relatively more in risky securities.

²For this and the following question, you may want to use the fact that if X is normally distributed with mean μ and variance σ^2 , then $(X - \mu)/\sigma$ is standard normally distributed.