EE/Ma 127c Error-Correcting Codes — R. J. McEliece
draft of May 30, 2001 — 162 Moore

Homework Assignment 6— **Final Version**
Due (in class) 9am June 1 , 2001

Reading:

Handout "Efficient Erasure Correcting Codes," pp. 569–572.

Problems to Hand In:

Problem 1. Consider the ensemble of LDPC codes with (left) degree profile $(\lambda_2, \lambda_3, \ldots, \lambda_N)$, and constant right degree a. If the ensemble rate is $R = 4/5$, what is the *smallest* possible value for a?

Problem 2. Let $H_1 \le \cdots \le H_n$ be a fixed list of real numbers, and let $p_1, \ldots, p_n$ be a corresponding probability density. Define

$$U = \langle H \rangle_p = \sum_i p_i H_i$$

$$S = \langle -\log p \rangle_p = -\sum_i p_i \log p_i.$$

For each value of U in the range $H_1 \le U \le H_n$, let $S_{\max}(U)$ denote the maximum possible value of S, i.e.,

$$S_{\max}(U) = \max\{-\sum_i p_i \log p_i : \sum_i p_i H_i = U\}.$$

(a) Show that the optimizing probabilites are of the form

$$p_i = p_i(\beta) = \frac{e^{-\beta H_i}}{Z(\beta)},$$

where $Z(\beta)$ (the partition function) is defined as

$$Z(\beta) = \sum_i e^{-\beta H_i}.$$

[Hint: If (p_i) is a density such that $\sum_i p_i H_i = U$, define $q_i = e^{-\beta H_i}/Z(\beta)$, and apply Jensen's inequality to the sum $\sum_i p_i \log(q_i/p_i)$.]

(b) Show that the relationship between U and $S_{\max}$ is given parametrically by the equations

$$U = -\frac{d}{d\beta} \log Z(\beta)$$

$$S_{\max} = \log Z(\beta) - \beta \frac{d}{d\beta} \log Z(\beta),$$

where the parameter β (the inverse temperature) runs from $-\infty$ to $+\infty$.

(c) For $(H_1, H_2, H_3) = (1, 2, 3)$, plot the function $S_{\max}$ as a function of U.