Homework Assignment 4, Solutions

Problem 1.

(a) In the RA code, for each information bit u, there are q repetition bits — $v_1, v_2, \cdots, v_q$ —that correspond to u in the block of encoded bits, where $v_i = u$ for $i = 1,2,\cdots,q$.

We can draw a junction tree, whose local domains and local kernels are listed below:

Local Domain	Local Kernel
$\{u\}$	*a priori* probability of *u*: $\Pr_{(a\_priori)}\{u=\varepsilon\}$, for $\varepsilon = 0,1$.
$\{u, v_1, v_2, \cdots, v_q\}$	$\chi(u, v_1, v_2, \cdots, v_q\} = \begin{cases} 1 \text{ } if \text{ } u = v_1 = v_2 = \cdots = v_q \\ 0 \quad otherwise \end{cases}$
$\{v_i\}$, for $i = 1,2,...,q$	Probability observed from channel: $\Pr\{v_i = \varepsilon \mid Y\}$ for $\varepsilon = 0,1$

In the junction tree, there is an edge between $\{u\}$ and $\{u, v_1, v_2, \cdots, v_q\}$, and there is an edge between $\{u, v_1, v_2, \cdots, v_q\}$ and $\{v_i\}$ for $i = 1,2,...,q$.

So

$$\Pr\{u = \varepsilon \mid evidence\}$$

$$= \left(\sum_{v_1, v_2, \cdots, v_q} \Pr\{v_1 \mid Y\} \Pr\{v_2 \mid Y\} \cdots \Pr\{v_q \mid Y\} \chi(u=\varepsilon, v_1, v_2, \cdots, v_q) \right) \Pr_{(a\_priori)}\{u = \varepsilon\}$$

$$= \Pr_{(a\_priori)}\{u=\varepsilon\} \prod_{i=1}^{q} \Pr\{v_i = \varepsilon \mid Y\}$$

Therefore,

$$\log \frac{\Pr\{u=0 \mid evidence\}}{\Pr\{u=1 \mid evidence\}} = \log \frac{\Pr_{(a\_priori)}\{u=0\} \prod_{i=1}^{q} \Pr\{v_i = 0 \mid Y\}}{\Pr_{(a\_priori)}\{u=1\} \prod_{i=1}^{q} \Pr\{v_i = 1 \mid Y\}}$$

$$= \log \frac{\Pr_{(a\_priori)}\{u=0\}}{\Pr_{(a\_priori)}\{u=1\}} + \sum_{i=1}^{q} \log \frac{\Pr\{v_i = 0 \mid Y\}}{\Pr\{v_i = 1 \mid Y\}} = LLR_{(a\_priori)} + \sum_{i=1}^{q} LLR_i$$

And clearly

$$\log \frac{\Pr\{v_i = 0 \mid evidence\}}{\Pr\{v_i = 1 \mid evidence\}} = \log \frac{\Pr\{u=0 \mid evidence\}}{\Pr\{u=1 \mid evidence\}} = LLR_{(a\_priori)} + \sum_{i=1}^{q} LLR_i \text{ since}$$

$v_i = u$, for $i = 1,2,\cdots,q$.

(b) By using the decoding rule in part (a), we get:

$$LLR_{u_1} = LLR_1^{(i)} + LLR_1^{(o)} + LLR_2^{(o)} + LLR_3^{(o)} = 0.4$$

$$LLR_{u_2} = LLR_2^{(i)} + LLR_4^{(o)} + LLR_5^{(o)} + LLR_6^{(o)} = -0.4$$

Problem 2.
Omitted.

Problem 3.
There are $(qk)!$ ways to place qk bits. However, permuting the q repetitions of any of the k information bits won't affect the result of the interleaving. So there are $\frac{(qk)!}{(q!)^k}$ ways for the interleaving.

I will give full score to anybody who gave $\frac{(qk)!}{(q!)^k}$ as the answer to this problem. However, strictly speaking that answer is not correct. Notice that by permuting the k information bits we won't change the vector space of the code. So actually there are $\frac{(qk)!}{(q!)^k k!}$ distinct (q,k) RA codes. For example, let's say $q=1$ and $k=2$. Then $\frac{(qk)!}{(q!)^k} = 2$, while $\frac{(qk)!}{(q!)^k k!} = 1$. There are two possible outputs of the interleaver, which are (u_1, u_2) and (u_2, u_1). It's easy to see those are actually the same code since they have the same vector space, and $\frac{(qk)!}{(q!)^k k!} = 1$ is the correctly answer.