EE/Ma 127c Error-Correcting Codes — R. J. McEliece
draft of May 16, 2001 — 162 Moore

Homework Assignment 4—
Due (in class) 9am May 18 , 2001

Reading:

Handout "The Serial Concatenation of Interleaved Codes . . . , " Secs. I, IIB, and V.

Problems to Hand In:

Problem 1. In class on Friday May 11, I discussed the iterative decoding of "serial"' turbo codes, with a structure like the one shown in Figure 14b of the handout "The Serial Concatenation of Interleaved Codes." I emphasized that the componet "APP" decoder for the outer code (the block labelled "MAP outer code" in Figure 14b) must be capable of producing APP's for the *encoded* bits as well as the information bits. I also gave as one example the "Repeat-Accumulate' code, in which the outer code is a simple "repeat each bit q times" device.

(a) Describe an efficient APP decoding rule for the information and encoded bits for a q-fold repetition code.

(b) Next consider the $(6, 2)$ code obtained by repeating each of the two information bits three times: in other words, the codeword associated with the information word (u_1, u_2) is $(u_1, u_1, u_1, u_2, u_2, u_2)$. Suppose the *a priori* log-likelihoods for the information bits (u_1, u_2) are

$$\mathrm{LLR}_1^{(i)} = 0.1, \quad \mathrm{LLR}_2^{(i)} = -0.1,$$

and the log-likelihoods observed from the channel of the 6 coded bits are

$$\mathrm{LLR}_1^{(o)} = 0.2, \quad \mathrm{LLR}_2^{(o)} = 0.1, \quad \mathrm{LLR}_3^{(o)} = 0.0,$$
$$\mathrm{LLR}_4^{(o)} = 0.0, \quad \mathrm{LLR}_5^{(o)} = -0.1, \quad \mathrm{LLR}_6^{(o)} = -0.2.$$

Compute the APP's (In log-likelihood form) for the two information bits and the six code bits.

Problem 2. In class Monday, May 14, I discussed a "linear congruential" method of generating random permutations of length $n = p - 1$, where p is an odd prime, i.e., those of the form

$$\pi(i) = ba^i (\mathrm{mod} p),$$

for $i = 1, 2, \ldots, n$. I stated that experiments with $p = 1103$ indicated that the choice $b = 1$ and $a = 127$ was better than e.g. $a = 3$, 5, 7. Ling Li has suggested that this may be due to correlations between $\pi(i)$ and $\pi(i + 1)$.

Test this suggestion by making a scatter plot of $\pi(i)$ vs. $\pi(i + 1)$ for $p = 1103$ and $a = 3, 5, 7, 9$, and 127, and comment. What about $\pi(i)$ vs. $\pi(i + 2)$, etc. ?

Problem 3. In class on Wednesday, May 16 I briefly discussed the problem of counting the number of distinct RA codes with parameters q and k, where q is the repetition number and k is the number of information bits. Taking into account the freedom of choosing the interleaver, how many distinct (q, k) RA codes are there? [Hint: the answer is *not* $(qk)!$.]