EE/Ma 127c Error-Correcting Codes R. J. McEliece
draft of April 25, 2001 162 Moore

Homework Assignment 3—**Final Version**
Due (in class) 9am April 27 , 2001

Reading:

Handout "The optimal decoding of linear codes for mimimizing symbol error rate."
handout: "Turbo decoding as an instance of . . . Belief propagation," Section IV.

Problems to Hand In:

Problem 1. Consider a finite-state (state set $\{0, 1, 2\}$) Markov chain $X_0, X_1, X_2, \ldots$, governed by the folllowing state transition matrix:

$$\begin{array}{c c} & \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \begin{pmatrix} 1/2 & 1/2 & 0 \\ 1/3 & 0 & 2/3 \\ 2/3 & 1/3 & 0 \end{pmatrix} \end{array}$$

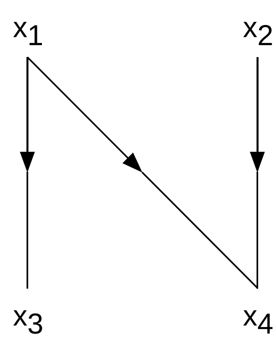
Suppose that the chain is initialzed with $X_0 = 0$. The successive terms X_1, X_2, X_3, X_4 are then transmitted through a noisy channel with transiton matrix

$$P = \begin{array}{c c} & \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \begin{pmatrix} .8 & .1 & .1 \\ .1 & .8 & .1 \\ .1 & .1 & .8 \end{pmatrix} \end{array}.$$

and observed to be $Y_1 = 1$, $Y_2 = 2$, $Y_3 = 2$, $Y_4 = 0$. Based on this evidence (call it $\mathcal{E}$), compute the *a posteriori* probabilities

$$\Pr\{X_i = j | \mathcal{E}\}, \qquad \text{for } i = 1, 2, 3, 4 \text{ and } j = 0, 1, 2.$$

Problem 2. In class on Monday April 23, I discussed the following simple Bayesian network:



In particular, I discussed the problem of inferring X_2 from X_3. However, after class several students (including Anxiao and Jeremy) pointed out to me that X_2 and X_3 are actually independent, so that observing X_3 tells us nothing about X_2! This problem is based on that mistake I made.

(a) Show that X_2 and X_3 are indeed independent, i.e., the joint density function $p(x_2, x_3)$ factors as $p(x_2)p(x_3)$.

(b) Which (if any) other pairs X_i, X_j from the given Bayesian network are independent?

(c) See if you can guess (not prove) an easy general way to predict which pairs of random variables in an arbitrary Bayesian network are independent.

Problem 3. let A be a $p \times q$ matrix, let B be a $q \times r$ matrix, and let C be a $r \times s$ matrix.

(a) How many scalar multiplications are required to compute the product AB?

(b) How many scalar multiplications are required to compute the product ABC when it is parenthesized as $(AB)C$? $A(BC)$?

(c) Suppose $p = 10$, $q = 100$, $r = 5$, and $s = 50$. What is the best way to compute ABC?