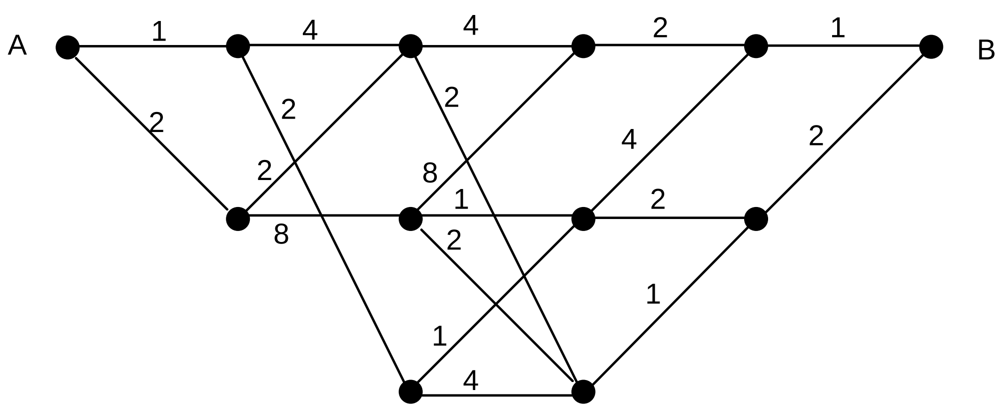
EE/Ma 127c Error-Correcting Codes R. J. McEliece
draft of April 16, 2001 162 Moore

Homework Assignment 2—**Final Version**
Due (in class) 9am April 20, 2001

Reading: Handout "The Forward-Backward Algorithm"
Handout "The optimal decoding of linear codes for mimimizing symbol error rate."
"The Generalized Distributive Law," (Esp. Page 326, column 2 – p. 327, col.1)

Problems to Hand In:

Problem 1. Consider the following weighted trellis, like the one I belabored in class on April 11.



(a) Compute the α_i's and β_i's.

(b) Find the value of the flow from A to B through each edge and vertex.

(c) Use the "log" forward-backward algorithm using the approximation $\log(x + y) \approx \max(\log x, \log y)$, to compute the same flows as in part (a).

Problem 2. In class on April 13, I showed that

$$\log(x + y) = \max(\log x, \log y) + f(\Delta),$$

where $f(\Delta) = \log(1 + e^{-\Delta})$, and $\Delta = |\log x - \log y|$. A two-bit approximation to $f(\Delta)$ is of the form

$$f(\Delta) \approx \begin{cases} y_1 & \text{if } 0 \le \Delta < \Delta_1 \\ y_2 & \text{if } \Delta_1 \le \Delta < \Delta_2 \\ y_3 & \text{if } \Delta_2 \le \Delta < \Delta_3 \\ y_4 & \text{if } \Delta_3 \le \Delta. \end{cases}$$

(Thus the approximation is characterized by the seven numbers $\Delta_1, \Delta_2, \Delta_3$, and y_1, y_2, y_3, y_4.) Find the "best" such approximation that you can. [You can define the goodness of the approximation in any way that seems reasonable to you. For example, you might want to minimize the maximum discrepancy between the true value of $F(\Delta)$ and its approximation.)

Problem 3. In class on April 16, I discussed the notion of a commutative semiring, and gave these five examples.

K	"$(+,0)$"	"$(\cdot,1)$"	short name
$[0,\infty)$	$(+,0)$	$(\cdot,1)$	sum-product (SP)
$(0,\infty]$	$(\min,\infty)$	$(\cdot,1)$	min-product (mP)
$[0,\infty)$	$(\max,0)$	$(\cdot,1)$	max-product (MP)
$(-\infty,+\infty]$	$(\min,\infty)$	$(+,0)$	min-sum (mS)
$[-\infty,\infty)$	$(\max,-\infty)$	$(+,0)$	max-sum (MS)

(a) Verify the distributive law, i.e.,

$$(a \cdot b) + (a \cdot c) = a \cdot (b + c),$$

for each of these 5 semirings.

(b) As I mentioned is class, the four semirings mP, MP, mS, and MS are all isomorphic to each other. For example, MP becomes mS under the mapping $x \mapsto -\log x$. Now complete the following isomorphism matrix:

$$\begin{array}{c c} & \begin{array}{cccc} mP & MP & mS & MS \end{array} \\ \begin{array}{c} mP \\ MP \\ mS \\ MS \end{array} & \begin{pmatrix} * & & & \\ & * & -\log x & \\ & & * & \\ & & & * \end{pmatrix} \end{array}$$