EE/Ma 127c Error-Correcting Codes

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Homework Assignment 1
Due (in class) 9am April 11, 2001

Reading: Johannesson and Zigangorov, Chapter 1, Section 1.5, pp. 23–24.
Johannesson and Zigangorov, Chapter 7, Section 7.1, pp. 317–321.
Handout “Near Shannon Limit ... Turbo-Codes” Sections I and II.
Handout “Turbo Codes for Deep Space Communications” pp. 29 – 30.
Handout “The Forward-Backward Algorithm”

Problems to Hand In:

Problem 1. (Repetition codes on the AWGN channel.) Consider using the $(n, 1)$ repetiton code on the AWGN channel. This code contains just two codewords, viz., $(+1, +1, \ldots, +1)$ and $(-1, -1, \ldots, -1)$. If $(x_1, x_2, \ldots, x_n)$ is the transmitted codeword, then $(y_1, y_2, \ldots, y_n)$ is received, where $y_i = x_i + z_i$, and $(z_1, z_2, \ldots, z_n)$ are i.i.d. Gaussians with mean zero and variance σ^2, where

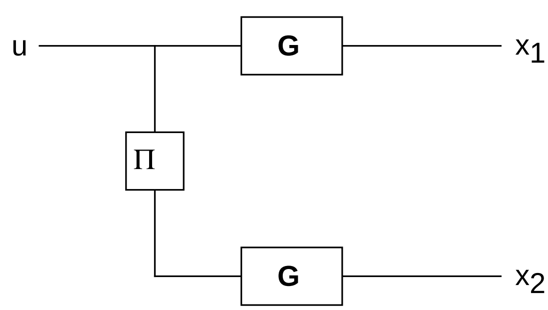
$$\sigma^2 = \left(\frac{2}{n}\frac{E_b}{N_0}\right)^{-1}.$$

(a) Describe a maximum-likelihood decoding algorithm for this code.

(b) Evaluate the decoder error probability (in terms of E_b/N_0) for the decoding algorithm you found in part a.

(c) Compare this performance to that of “uncoded BPSK,” which was described in class on April 2.

Problem 2. Consider a simple “turbocode” of the following type:



Suppose $u = (u_1, u_2, , u_3, u_4)$ and G is the following 4×4 matrix:

$$G = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix}.$$

Then the overall code is an $(8, 4)$ binary linear code. However, there are $4! = 24$ choices for the interleaver. The problem: find a “best” interleaver, i.e., one that :

(1) . Ensures that the overall code has dimension 4.

(2) . Maximizes the minimum distance of the code.

Problem 3. Consider the $(4, 2)$ binary linear code with generator matrix

$$G = \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{pmatrix}.$$

Thus the information block (u_1, u_2) is encoded as (x_1, x_2, x_3, x_4), where $x_1 = u_1$, $x_2 = u_2$, $x_3 = u_1 + u_2$, and $x_4 = u_1 + u_2$. Suppose that an unknown codeword is transmitted over the following DMC:

$$\begin{array}{c c} & \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{pmatrix} 1/2 & 1/4 & 1/8 & 1/8 \\ 1/8 & 1/8 & 1/4 & 1/2 \end{pmatrix} \end{array}.$$

(a) Suppose the *a priori* input distribution is

$$p^0(0) = p^0(1) = 1/2,$$

and ther received word is $ABCD$. Compute the *a posteriori* probabilities for u_1 and u_2 in log-likelihood form. What is the "extrinsic information" (in log-likelihood form) for u_1 and u_2?

(b) Now suppose the *a priori* input distribution is

$$p^0(0) = 1/3, \quad p^0(1) = 2/3$$

and the received word is still $ABCD$. Answer the same questions.

Problem 4. In class on April 9, I proved thatif u, v, and x are vertices in a trellis, that

$$\mu_x(u, v) = \mu(u, x)\mu(x, v).$$

(a) Now suppose that u and v are vertices, and e is an edge. Prove that

$$\mu_e(u, v) = \mu(u, x)w(e)\mu(y, v),$$

where x is the initial vertex and y is the final vertex of e.

(b) Discuss the computational savings implied by the result of part (a).