BEM 103 Introduction to Finance - Final Exam

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1. Let the cash flow at year t be C_t:

$$\begin{aligned} C_0 &= -240 - 60 = -300, \\ C_1 = C_2 &= (200 - 100)(1 - 40\%) + \frac{240}{3} \times 40\% = 92, \\ C_3 &= C_2 + 40(1 - 40\%) + 60 = 176. \end{aligned}$$

Hence,

$$\text{NPV} = \sum_{t=0}^{3} \frac{C_t}{(1+8\%)^t} \approx 3.775.$$

2. For the first two years, the accounting profit is $200 - 100 - \frac{240}{3} = 20$ each. Thus the tax shield by the coupon issuing is only

$$20 \times 40\% = 8.$$

For the third year, the accounting profit is $20 + 40 = 60$. Thus the tax shield is

$$25 \times 40\% = 10.$$

The project's NPV changes due to the tax shields, that is,

$$\text{NPV} \approx 3.775 + \frac{8}{1.08} + \frac{8}{1.08^2} + \frac{10}{1.08^3} \approx 25.979.$$

3. The state-price probability of the "up" state is

$$p^u = \frac{1 + r - d}{u - d} = 0.46,$$

and that of the "down" state is $p^d = 1 - p^u = 0.54$.

- When the state at $t = 1$ is "up," the value of one call option with $K = 100$ is

$$C_1^u = \frac{1}{1+r}\left[p^u \cdot \max(100u^2 - 100, 0) + p^d \cdot \max(100ud - 100, 0)\right] \approx 54.762;$$

while the value of one put option with $K = 100$ is

$$P_1^u = \frac{1}{1+r}\left[p^u \cdot \max(100 - 100u^2, 0) + p^d \cdot \max(100 - 100ud, 0)\right] = 0.$$

- When the state at $t = 1$ is "down," the value of the call option is

$$C_1^d = \frac{1}{1+r}\left[p^u \cdot \max(100du - 100, 0) + p^d \cdot \max(100d^2 - 100, 0)\right] = 0;$$

while the value of the put option is

$$P_1^d = \frac{1}{1+r}\left[p^u \cdot \max(100 - 100du, 0) + p^d \cdot \max(100 - 100d^2, 0)\right] \approx 28.571.$$

With the compound option, we can choose the option with higher value at each state. That is, we can choose the call option at the "up" state and the put option at the "down" state. Thus the value of the compound option is

$$V_0 = \frac{1}{1+r}\left[p^u \cdot C_1^u + p^d \cdot P_1^d\right] \approx 38.685.$$

4. Let the state-price probability of the "up" state be p^u. From

$$1500p^u + 800(1 - p^u) = 1000(1 + 5\%),$$

we get $p^u = \frac{5}{14}$.
At the "up" state, if the bond is not converted, the pay off of one bond is 10 and the value of one share of equity is

$$\frac{1500 - 10 \times 100}{100} = 5 < 10.$$

Hence we'd better not convert the bond. At the "down" state, the firm has value 800, less than the total face value of the bond. Thus the total bond pay off is 800. Hence the value of the bond today is

$$B = \frac{1}{1+r}\left[p^u \times 1000 + (1 - p^u) \times 800\right] \approx 829.932.$$

By value additivity, the value of the equity is

$$E = 1000 - B \approx 170.068.$$

5. Let τ_G be the capital gains tax rate and τ_D be the ordinary income (including dividends) tax rate. $P_B = 30$ is the stock price just before the "ex" day, $P_A = 27$ is the stock price just after the "ex" day, $P_0 = 20$ is the original stock price, and $D = 3$ is the dividend amount. The after-tax wealth under the first scenario (the firm pays dividends) is

$$100D(1 - \tau_D) + 100P_A - 100\tau_G(P_A - P_0) = 3000 - 300\tau_D - 700\tau_G;$$

and that under the second scenario (the firm doesn't pay dividends) is

$$100P_B - 100\tau_G(P_B - P_0) = 3000 - 1000\tau_G.$$

(a) If $\tau_G = \tau_D = 28\%$, then the after-tax wealth is always

$$3000 - 1000 \times 28\% = 2720.$$

(b) If $\tau_G = 16\%$ and $\tau_D = 40\%$, then for the first scenario, the wealth is

$$3000 - 300 \times 40\% - 700 \times 16\% = 2768;$$

and for the second scenario, the wealth is

$$3000 - 1000 \times 16\% = 2840.$$

6. They should have made 20. Let the values of the bond, equity, and the project be B, E, and V, respectively. Let I be the initial investment needed to start the project. Hence the "remaining funds" the owners provided is $(I - B)$. Since we have $\text{NPV} = V - I$ and $V = E + B$, what they earned should be

$$E - (I - B) = E + B - I = V - I = \text{NPV} = 20.$$

7. I suggest

- Firm A issues stocks of value B;
- Firm B borrows B as debt from firm A, and use the money to buy back some of its stocks;
- Firm B will pay dividends to firm A; A then uses the same money to pay dividends to its bond holders.

Thus the dividends payment is transferred from firm A to firm B. If we look at the two firms as a whole, the cash flows do not change, except that firm B now gains tax shield from the dividends, which is $\$5 \times 20\% = \$1$. Thus the total value of the two firms is increased.

8. Let R_t be the return over year t. The wealth at the end of T years is

$$W_T = \left(\prod_{t=1}^{T} R_t\right) \cdot W_0.$$

The percentage change in the wealth over T years is

$$\ln W_T - \ln W_0 = \sum_{t=1}^{T} \ln R_t.$$

Since $\ln R_t$'s are i.i.d. $\sim \mathcal{N}(0.15, 0.15^2)$, the expected percentage change over T years is

$$\mu_T = \sum_{t=1}^{T} E\left[\ln R_t\right] = 0.15T$$

and the variance is

$$\sigma_T^2 = \sum_{t=1}^{T} \sigma^2\left[\ln R_t\right] = 0.15^2 T,$$

or, the standard deviation is $\sigma_T = 0.15\sqrt{T}$. Note that while the expectation μ_T increases linearly with T, the standard deviation is only proportional to $\sqrt{T}$.

For $T = 1$, $\mu_1 = 0.15$ and $\sigma_1^2 = 0.15^2$ ($\sigma_1 = 0.15$); for $T = 15$, $\mu_{15} = 2.25$ and $\sigma_{15}^2 = 0.3375$ ($\sigma_{15} \approx 0.581$).

9. The probability that $W_T > W_0$ is

$$\begin{aligned}
\Pr\left\{\sum_{t=1}^{T} \ln R_t > 0\right\} &= \int_0^{\infty} \frac{1}{\sqrt{2\pi\sigma_T^2}} e^{-\frac{(x-\mu_T)^2}{2\sigma_T^2}}\, dx \\
&= \int_{-\frac{\mu_T}{\sigma_T}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}\, dx \\
&= \frac{1}{2} + \int_0^{\sqrt{T}} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}\, dx.
\end{aligned}$$

The bigger T is, the higher the probability is. So the probability that you end up with more than you started over one year is *lower* than that over 15 years.

10. The probability is $\Pr\{W_T < 15\% \cdot W_0\}$, or

$$\begin{aligned}
\Pr\left\{\sum_{t=1}^{T} \ln R_t < \ln 0.15\right\} &= \int_{-\infty}^{\ln 0.15} \frac{1}{\sqrt{2\pi\sigma_T^2}} e^{-\frac{(x-\mu_T)^2}{2\sigma_T^2}}\, dx \\
&= \int_{-\infty}^{\frac{\ln 0.15-\mu_T}{\sigma_T}} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}\, dx.
\end{aligned}$$

For $T = 1$,

$$\frac{\ln 0.15 - \mu_T}{\sigma_T} \approx -13.6475,$$

while for $T = 15$,

$$\frac{\ln 0.15 - \mu_T}{\sigma_T} \approx -7.1385.$$

Thus the probability over one year that you end up with less than 15% of your initial wealth is lower than that over 15 years.