Outline

- From training to testing
- Illustrative examples
- Key notion: break point
- Puzzle
Example 1: positive rays

\[ h(x) = -1 \]

\( x_1 \quad x_2 \quad x_3 \quad \ldots \quad x_N \)

\[ h(x) = +1 \]

\( a \)

\( \mathcal{H} \) is set of \( h: \mathbb{R} \rightarrow \{-1, +1\} \)

\[ h(x) = \text{sign}(x - a) \]

\[ m_{\mathcal{H}}(N) = N + 1 \]
Example 2: positive intervals

\[ h(x) = -1 \]
\[ h(x) = +1 \]
\[ h(x) = -1 \]

\( x_1 \quad x_2 \quad x_3 \quad \ldots \quad x_N \)

\[ \mathcal{H} \text{ is set of } h: \mathbb{R} \rightarrow \{-1, +1\} \]

Place interval ends in two of \( N + 1 \) spots

\[ m_{\mathcal{H}}(N) = \left( \frac{N+1}{2} \right) + 1 = \frac{1}{2} N^2 + \frac{1}{2} N + 1 \]
Example 3: convex sets

\( \mathcal{H} \) is set of \( h: \mathbb{R}^2 \rightarrow \{-1, +1\} \)

\( h(\mathbf{x}) = +1 \) is convex

\( m_\mathcal{H}(N) = 2^N \)

The \( N \) points are 'shattered' by convex sets
The 3 growth functions

• $\mathcal{H}$ is positive rays:

\[ m_{\mathcal{H}}(N) = N + 1 \]

• $\mathcal{H}$ is positive intervals:

\[ m_{\mathcal{H}}(N) = \frac{1}{2}N^2 + \frac{1}{2}N + 1 \]

• $\mathcal{H}$ is convex sets:

\[ m_{\mathcal{H}}(N) = 2^N \]