Error measures

What does “$h \approx f$” mean?

Error measure: $E(h, f)$

Almost always pointwise definition: $e(h(x), f(x))$

Examples:

- Squared error: $e(h(x), f(x)) = (h(x) - f(x))^2$
- Binary error: $e(h(x), f(x)) = [h(x) \neq f(x)]$
From pointwise to overall

Overall error $E(h, f) = \text{average of pointwise errors } e(h(x), f(x))$.

In-sample error:

$$E_{\text{in}}(h) = \frac{1}{N} \sum_{n=1}^{N} e(h(x_n), f(x_n))$$

Out-of-sample error:

$$E_{\text{out}}(h) = \mathbb{E}_x[e(h(x), f(x))]$$
The learning diagram - with pointwise error

**UNKNOWN TARGET FUNCTION**

\[ f : \mathcal{X} \rightarrow \mathcal{Y} \]

**HYPOTHESIS SET**

\[ \mathcal{H} \]

**TRAINING EXAMPLES**

\[ (x_1, y_1), \ldots, (x_N, y_N) \]

**LEARNING ALGORITHM**

\[ \mathcal{A} \]

**PROBABILITY DISTRIBUTION**

\[ P \text{ on } \mathcal{X} \]

**FINAL HYPOTHESIS**

\[ g : \mathcal{X} \rightarrow \mathcal{Y} \]

\[ g(x) \approx f(x) \]

\[ x_1, \ldots, x_N \]

\[ x \]

\[ \mathcal{P} \times \mathcal{X} \]
How to choose the error measure

Fingerprint verification:

Two types of error:

\textit{false accept} and \textit{false reject}

How do we penalize each type?

\[ f \]

<table>
<thead>
<tr>
<th>( h )</th>
<th>(+1)</th>
<th>(-1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>no error</td>
<td>false accept</td>
<td>false reject</td>
</tr>
<tr>
<td>you</td>
<td>intruder</td>
<td></td>
</tr>
</tbody>
</table>
The error measure - for supermarkets

Supermarket verifies fingerprint for discounts

False reject is costly; customer gets annoyed!

False accept is minor; gave away a discount and intruder left their fingerprint 😊

<table>
<thead>
<tr>
<th>$h$</th>
<th>$f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>+1</td>
<td>+1</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>−1</td>
<td>10</td>
</tr>
</tbody>
</table>

\[ f \rightarrow \begin{cases} +1 & \text{you} \\ −1 & \text{intruder} \end{cases} \]
The error measure for the CIA

CIA verifies fingerprint for security

False accept is a disaster!

False reject can be tolerated
Try again; you are an employee 😊

\[
\begin{array}{c|cc}
  h & f & \\
  +1 & +1 & -1 \\
  -1 & 0 & 1000 \\
  \\
\end{array}
\]

\[
f \rightarrow \begin{cases} 
  +1 & \text{you} \\
  -1 & \text{intruder} 
\end{cases}
\]
Take-home lesson

The error measure should be specified by the user.

Not always possible. Alternatives:

Plausible measures: squared error \(\equiv\) Gaussian noise

Friendly measures: closed-form solution, convex optimization
The learning diagram - with error measure

**UNKNOWN TARGET FUNCTION**
\[ f : X \rightarrow Y \]

**TRAINING EXAMPLES**
\[ (x_1, y_1), \ldots, (x_N, y_N) \]

**LEARNING ALGORITHM**
\[ \mathcal{A} \]

**HYPOTHESIS SET**
\[ \mathcal{H} \]

**ERROR MEASURE**
\[ e(\cdot) \]

**PROBABILITY DISTRIBUTION**
\[ P \text{ on } X \]

**FINAL HYPOTHESIS**
\[ g : X \rightarrow Y \]

**TRAINING EXAMPLES**
\[ x, x_1, \ldots, x_N \]

**ERROR MEASURE**
\[ g(x) \approx f(x) \]