A simple hypothesis set - the ‘perceptron’

For input $\mathbf{x} = (x_1, \cdots, x_d)$ ‘attributes of a customer’

- Approve credit if $\sum_{i=1}^{d} w_i x_i > \text{threshold}$,
- Deny credit if $\sum_{i=1}^{d} w_i x_i < \text{threshold}$.

This linear formula $h \in \mathcal{H}$ can be written as

$$h(\mathbf{x}) = \text{sign} \left( \left( \sum_{i=1}^{d} w_i x_i \right) - \text{threshold} \right)$$
\[ h(\mathbf{x}) = \text{sign} \left( \left( \sum_{i=1}^{d} w_i x_i \right) + w_0 \right) \]

Introduce an artificial coordinate \( x_0 = 1 \):

\[ h(\mathbf{x}) = \text{sign} \left( \sum_{i=0}^{d} w_i x_i \right) \]

In vector form, the perceptron implements

\[ h(\mathbf{x}) = \text{sign}(\mathbf{w}^\top \mathbf{x}) \]
A simple learning algorithm - PLA

The perceptron implements
\[ h(x) = \text{sign}(w^T x) \]

Given the training set:
\[ (x_1, y_1), (x_2, y_2), \ldots, (x_N, y_N) \]

pick a misclassified point:
\[ \text{sign}(w^T x_n) \neq y_n \]

and update the weight vector:
\[ w \leftarrow w + y_n x_n \]
Iterations of PLA

- One iteration of the PLA:

  \[ \mathbf{w} \leftarrow \mathbf{w} + yx \]

  where \((x, y)\) is a misclassified training point.

- At iteration \(t = 1, 2, 3, \ldots\), pick a misclassified point from

  \( (x_1, y_1), (x_2, y_2), \ldots, (x_N, y_N) \)

  and run a PLA iteration on it.

- That's it!